**Title: Variance Reduction for Monte Carlo Option Pricing**

**Abstract**

In the financial services industry, an option is a financial product that gives the holder the right (but not the obligation) to buy or sell another (underlying) asset. Options are valuable assets for both price speculation and risk management; therefore, efficiently computing the most accurate and low-error option price is a lucrative problem for banks and financial institutions. Monte Carlo simulation is a widely used method in computational finance for pricing complex financial derivatives, particularly when closed-form solutions are unavailable. However, its efficiency can be severely hampered by high variance, leading to slow convergence and inaccurate estimates. While several variance reduction techniques have been proposed, their relative effectiveness remains problem-specific and often lacks comprehensive benchmarking across different option types and market regimes.

In this paper, we evaluate a suite of variance reduction techniques specifically for the arithmetic Asian style of option on a Geometric Brownian motion model (whose distribution does not have a closed form solution), and propose a combination of techniques that best reduce the variance of the Monte Carlo estimator.

**Introduction**

Options are financial derivatives whose value depends on the behavior of an underlying asset over a fixed time horizon. More formally, let V denote the price of an option whose value at maturity depends on a stochastic process S\_t, typically modeled as Geometric Brownian Motion (GBM) under the Black–Scholes framework. The option payoff is generally expressed as V = max(f(S) - K, 0), where K is the strike price and f(S) is a function specific to the option style. For example, European options take f(S) = S\_T, Asian options use the average price over time, i.e., f(S) = (1/T)∫₀ᵀ S\_t dt, and barrier options depend on whether the path of S\_t crosses a predefined barrier level during the contract life (Hull, 2015; Glasserman, 2004).

For many such contracts, particularly those with path-dependent payoffs like Asian or barrier options, a closed-form analytical solution is not available. In such cases, Monte Carlo (MC) methods provide a flexible numerical approach to approximate the risk-neutral price of the option v = E^Q[V], by simulating a large number of sample paths of the underlying asset and averaging the discounted payoffs (Boyle, 1977; Glasserman, 2004). The stochastic nature of these methods introduces statistical error: the convergence rate of standard Monte Carlo is O(N^{-1/2}), where N is the number of paths. This implies that reducing the standard error by a factor of 10 requires increasing the sample size by a factor of 100, which can be computationally expensive for complex contracts.

As such, a key concern in Monte Carlo option pricing is the variance of the estimator. High-variance estimators require more simulations to reach a given accuracy, which may be infeasible in time-sensitive applications. Variance reduction techniques aim to decrease this variance without introducing bias, thereby improving computational efficiency while maintaining pricing accuracy (Caflisch et al., 1997; Glasserman et al., 1999).

Among the various option styles, Asian options are particularly important due to their use in markets to reduce price manipulation and volatility exposure. Their path-average structure makes them computationally intensive and analytically intractable, especially for arithmetic averaging, for which no closed-form solution exists. Monte Carlo methods are thus the standard tool for pricing such options, but are only practical when enhanced with techniques like control variates, antithetic variates, or quasi-Monte Carlo methods (Kemna & Vorst, 1990; Fu et al., 1999).

Despite its broad applicability, standard Monte Carlo methods suffer from slow convergence and high variance, particularly for Asian options with long monitoring periods or deep in-/out-of-the-money strikes. To address this inefficiency, significant research has focused on **variance reduction techniques**. Among the most effective is the **control variate method**, which leverages the exact solution for the geometric-average Asian option—available under Black–Scholes assumptions—to substantially reduce variance when pricing the arithmetic-average counterpart [Kemna & Vorst, 1990; Fu et al., 1999]. Other approaches include **antithetic variates or** **importance sampling**, each offering unique benefits depending on the option’s moneyness and monitoring frequency [Glasserman, Heidelberger, & Shahabuddin, 1999].

In parallel, **Quasi-Monte Carlo (QMC)** methods have gained prominence as an alternative to standard Monte Carlo. By using low-discrepancy sequences such as Sobol’ or Halton, QMC techniques achieve faster convergence rates in practice, often approaching O(N-1) instead of the traditional O(N-1/2) [Caflisch et al., 1997]. When combined with path-construction strategies like Brownian bridge or principal component analysis (PCA), QMC methods can significantly outperform classical simulations for high-dimensional path-dependent options such as Asians [Imai & Tan, 2006; Sabino, 2007].

This paper investigates and compares several variance reduction methods for pricing Asian options using Monte Carlo simulation. Through theoretical discussion and empirical experiments, we assess the impact of each method on estimator accuracy and computational efficiency. The ultimate goal is to identify combinations of techniques that enable fast and reliable pricing for path-dependent options where standard methods fall short.

**Methodology**

Of the methods mentioned in the literature, we will explore a subset of variance reduction methods and evaluate their performance for pricing an arithmetic Asian option.

Antithetic Variance

Control Variates

Importance Sampling

Quasi Monte Carlo

**Results**

**Discussion**

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Appendix